Data assimilation with Lorenz 3-variable model

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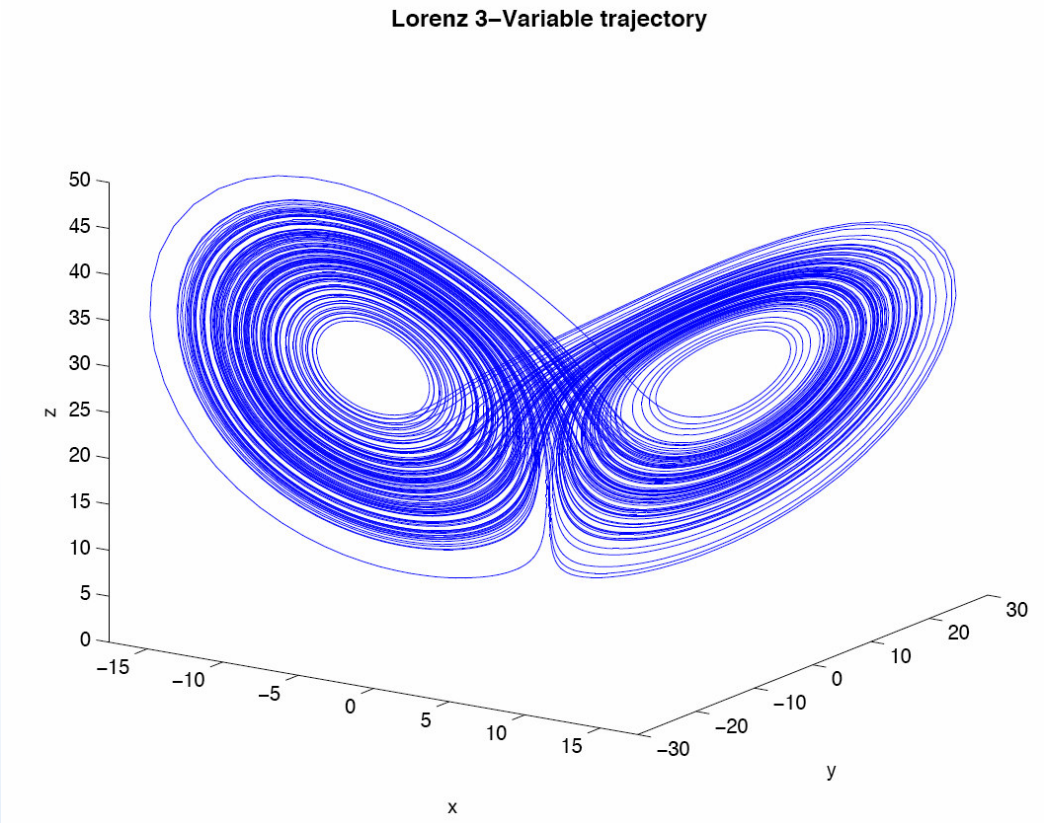
Modified by Juan Ruiz.

Governing equations



$$\frac{dx}{dt} = \sigma(y - x)$$

$$\frac{dy}{dt} = rx - y - xz$$

$$\frac{dz}{dt} = xy - bz$$

Lorenz, E. N, 1963: Deterministic nonperiodic flow. *J. Atmos Sci.* **20**, 130-141.

Optimal interpolation scheme

OI estimates the analysis state with the available observations and short-range forecast (background state), given the **time-independent** error covariance.

Analysis equation

$$\mathbf{x}_a = \mathbf{x}_b + \mathbf{W}(\mathbf{y}_o - H(\mathbf{x}_b))$$

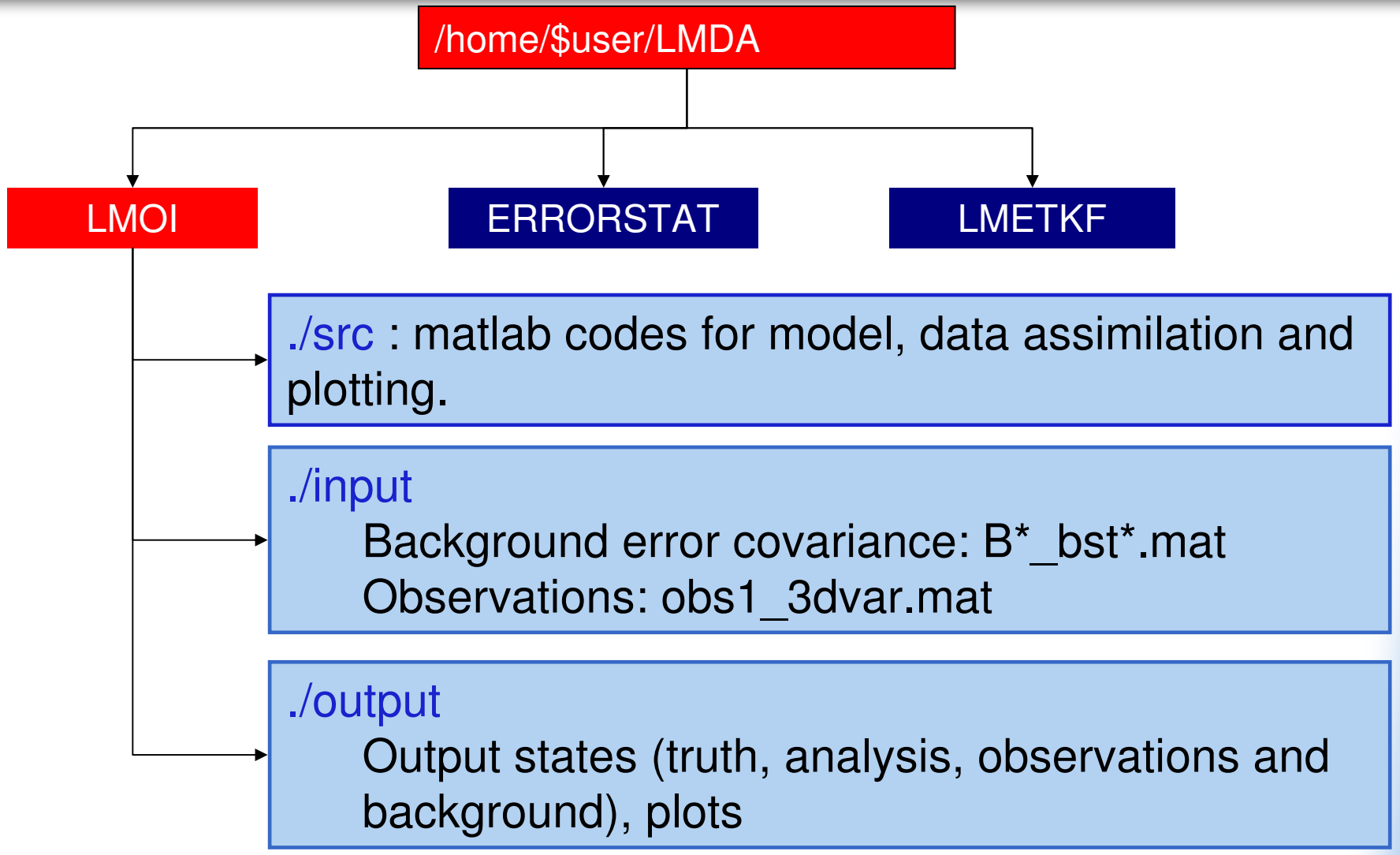
$$\mathbf{W} = \mathbf{B}\mathbf{H}^T(\mathbf{R}+\mathbf{H}\mathbf{B}\mathbf{H}^T)^{-1}$$

$$\mathbf{B} = \overline{(x_b - x_t)(x_b - x_t)^T};$$

$$\mathbf{R} = R_0 \times \mathbf{I}$$

$\mathbf{x}_b$:background state
$\mathbf{x}_a$: analysis state
$\mathbf{y}_o$: observations
$\mathbf{H}$: observation operator
$\mathbf{B}$: background error covariance
(averaged from the true background error collected for a very long period.
$\mathbf{R}$: observation error covariance

Package


/home/$user/LMDA
LMOI
ERRORSTAT
LMETKF
./src : matlab codes for model, data assimilation and plotting.
./input
Background error covariance: B*_bst*.mat
Observations: obs1_3dvar.mat
./output
Output states (truth, analysis, observations and background), plots

Matlab codes (I)

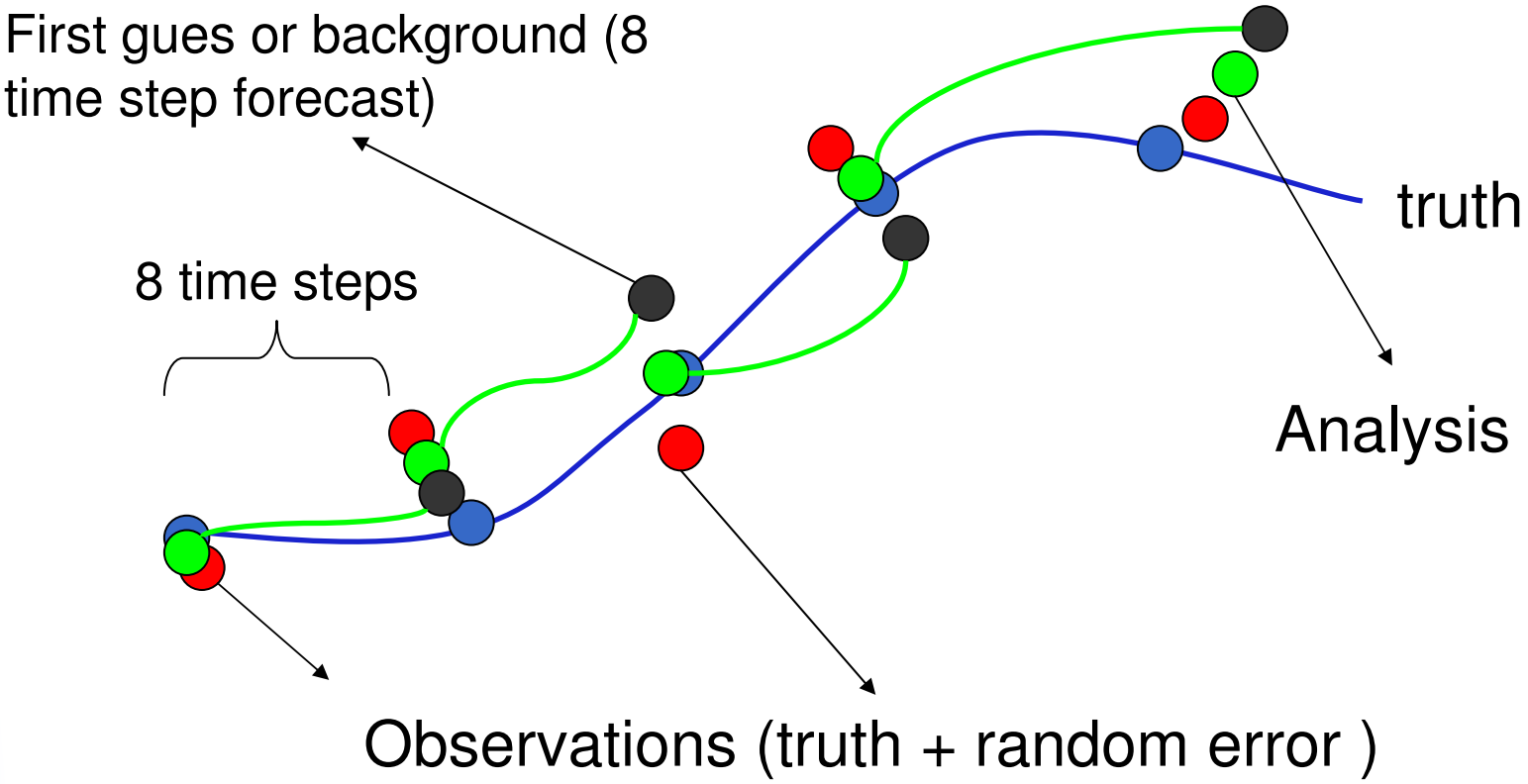
- main_drive.m: **main driver**

  Parameters control model and data assimilation:

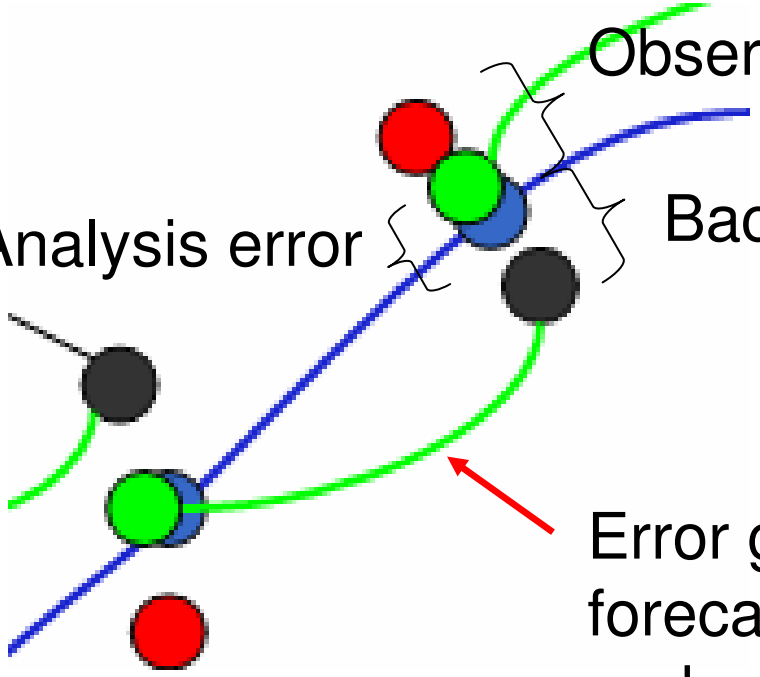
  1. bst: Observation/analysis interval (default=8)
  2. $\mathbf{R}_0$: Observation error variance
  3. iobs: observation locations (default [1;2;3], observing all variables)
  4. (1) and (3) determine which file for background error covariance to load. (ex: With 8 time-step analysis interval and observing x and y, the corresponding file is **Bxy_bst8.mat**)

To run the code in type main_drive in Matlab under ./src

DA cycle


First gues or background (8 time step forecast)
8 time steps
truth
Analysis
Observations (truth + random error)

DA cycle


Observation error
Background error
Analysis error
Error growth during the forecast due to inestabilities and model error.

Matlab codes (II)

- Model

  L63eqs.m: governing equations

  stepit.m: forward integration by Runge-Kutta method

- DA procedure

  DA_init.m: initialize matrix operators (**B**, **R**, and **H**) for OI

  RUN_OI.m: compute OI analysis

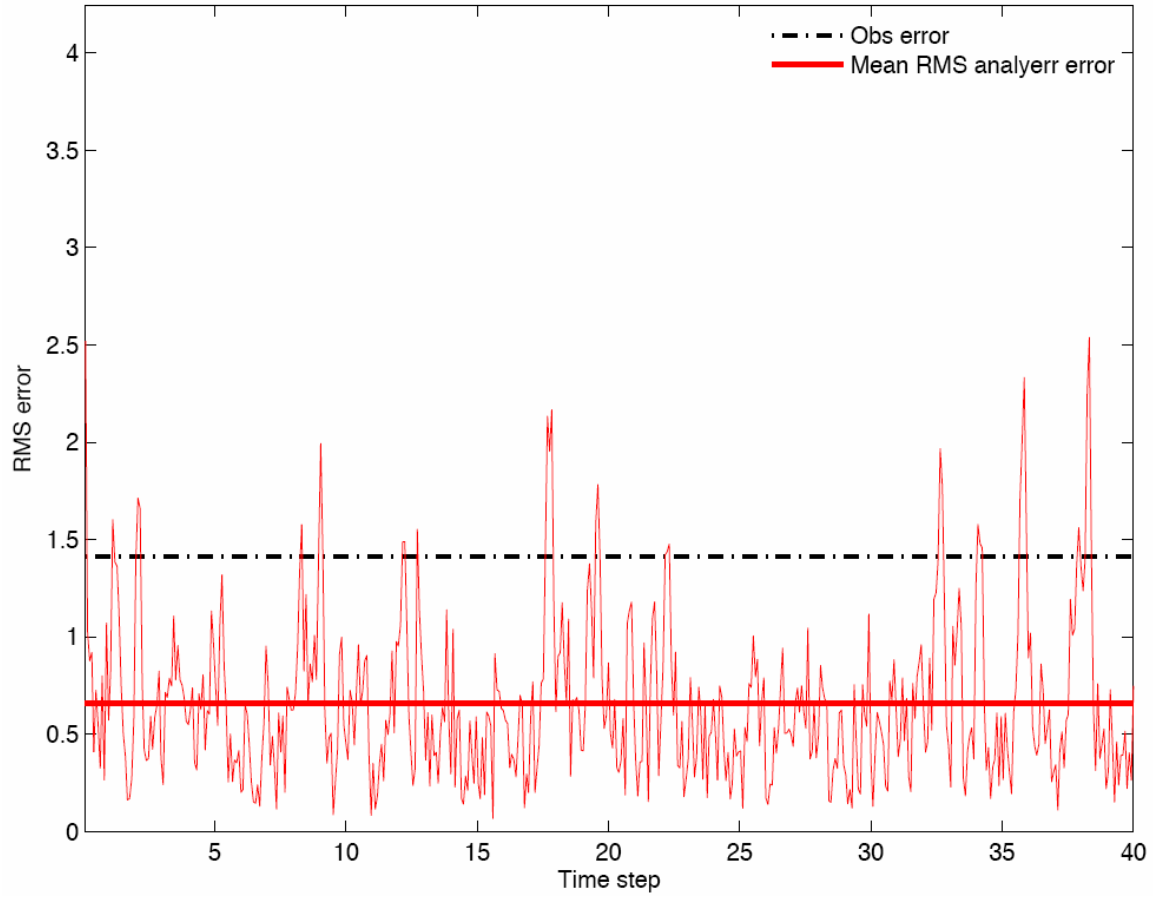
- Plotting tools

  Daerrplt.m: plot the RMS analysis error

  stateplt.m: plot the analysis against truth by variable

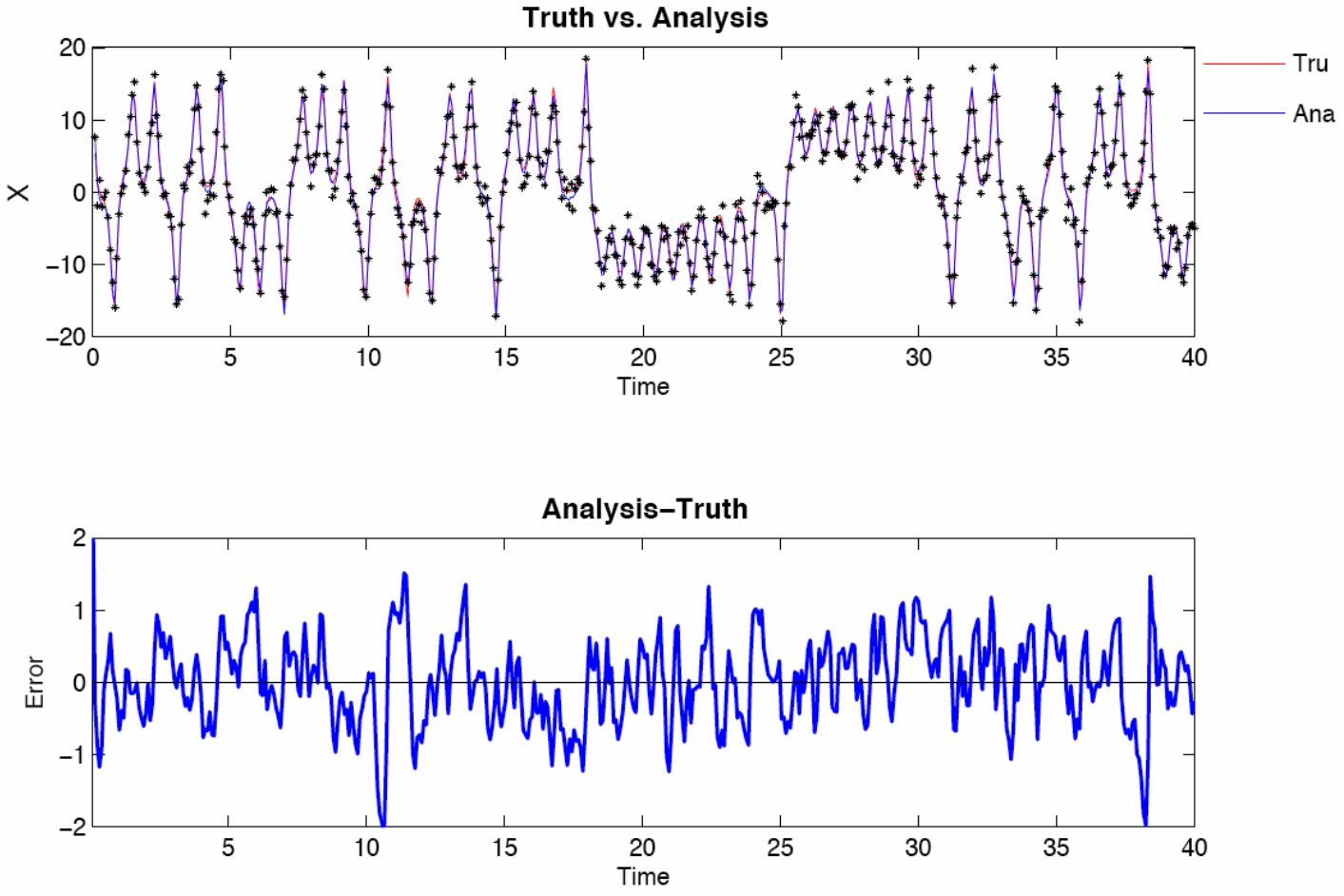
Example of OI analysis error

(observing x,y,z every 8 time-step)



Example: analysis vs. truth

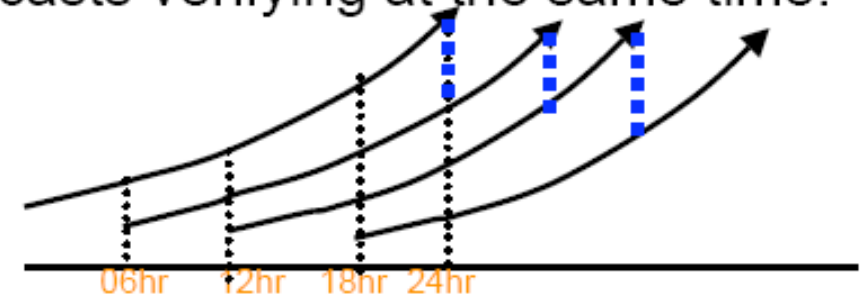
for x variable (observing x,y,z every 8 time-step)



Background error covariance estimation

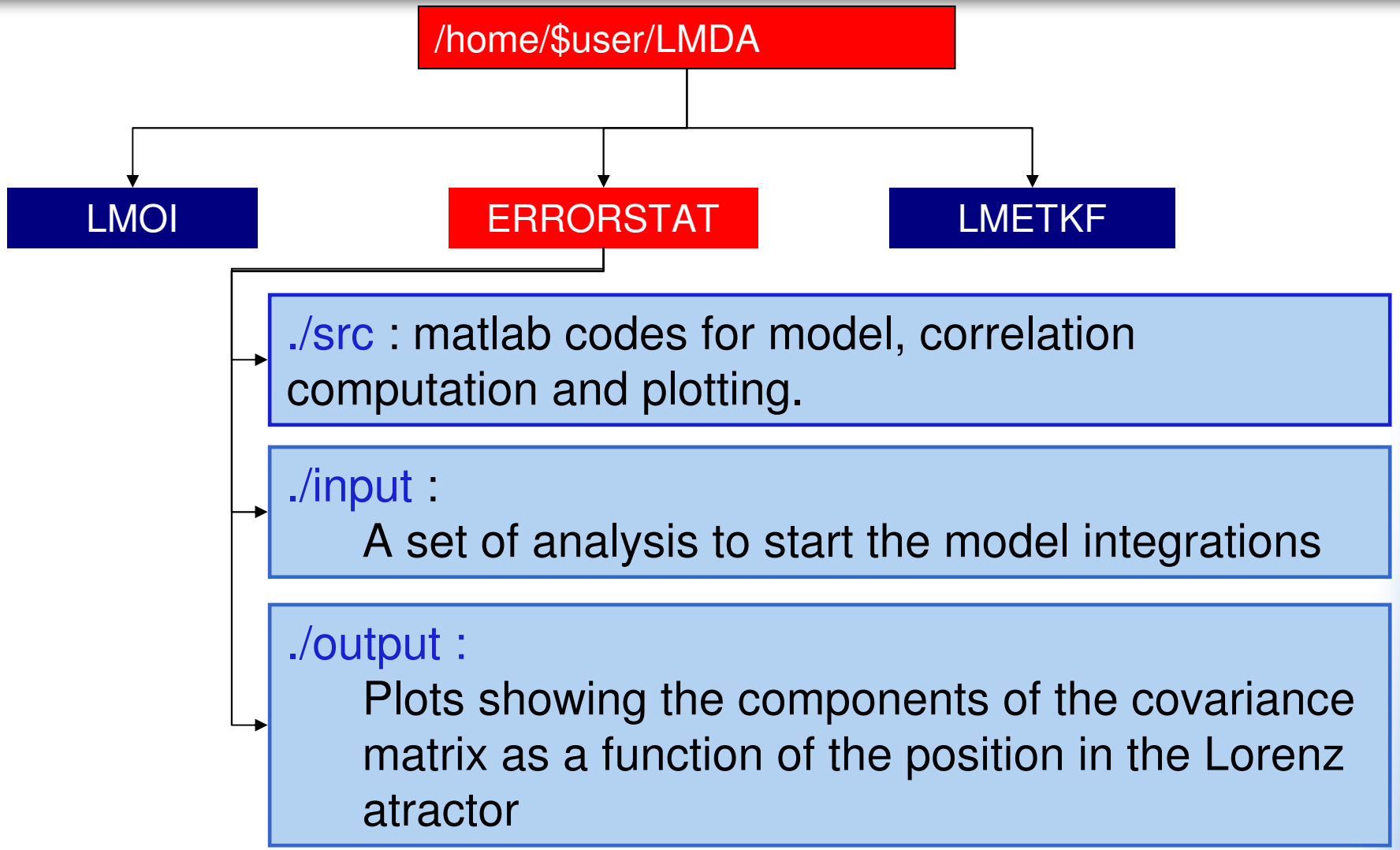
The NMC method (Parish and Deber, 1992)

The structure of the forecast error covariance is estimated as the average over many differences between two short-range model forecasts verifying at the same time.



In the case of the 3 variable model, we need to estimate the error standard deviation for each variable and the error covariance between variables.

Package


/home/$user/LMDA
LMOI
ERRORSTAT
LMETKF
./src : matlab codes for model, correlation computation and plotting.
./input :
A set of analysis to start the model integrations
./output :
Plots showing the components of the covariance matrix as a function of the position in the Lorenz atractor

Matlab codes (III)

- Error covariance estimation

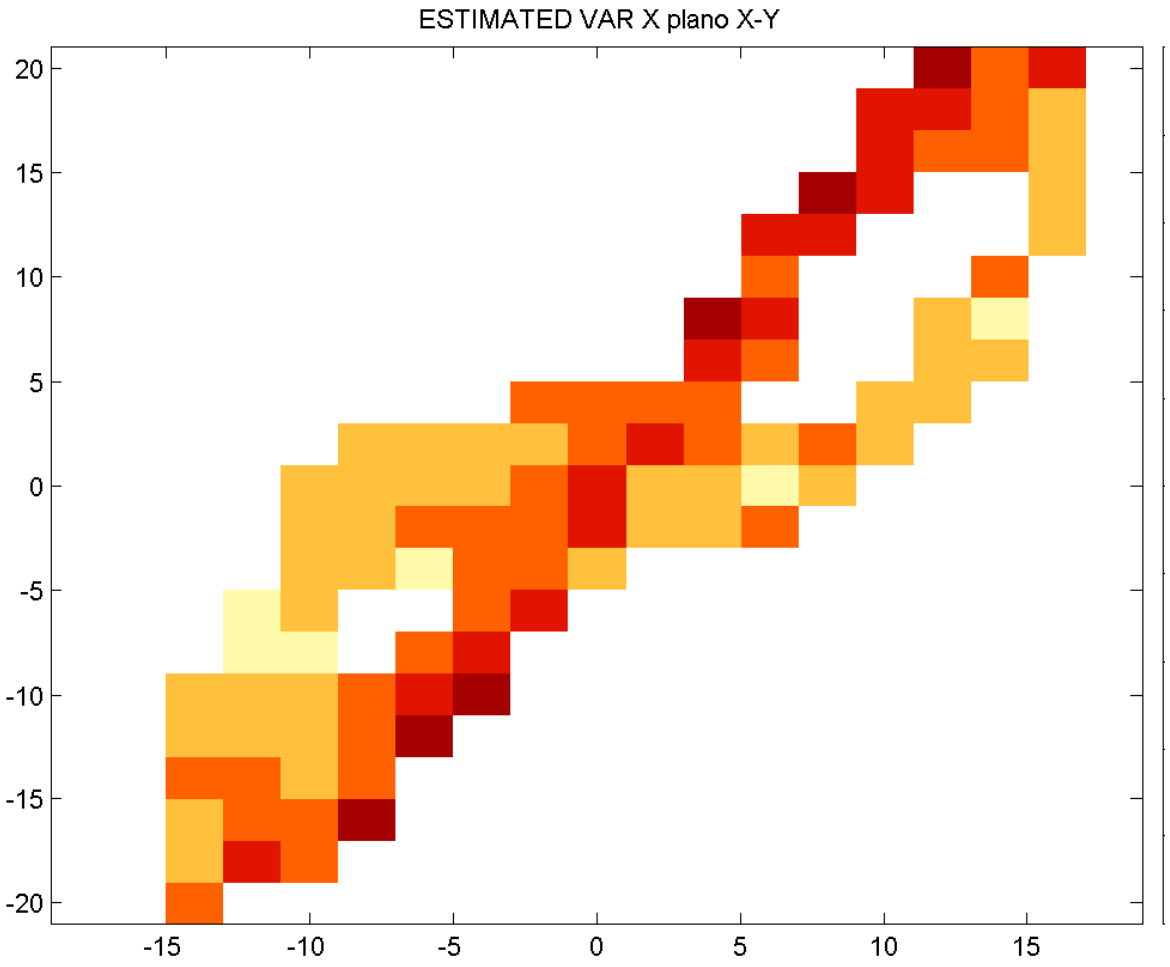
main_drive.m: Integrates the model for 8 and 16 time steps starting from the analysis available in the input folder. Estimates the background error as the difference between the 16 and 8 time steps forecasts.

correlacion.m: Computes the correlation between two variables as a function of the position on the Lorenz atractor.

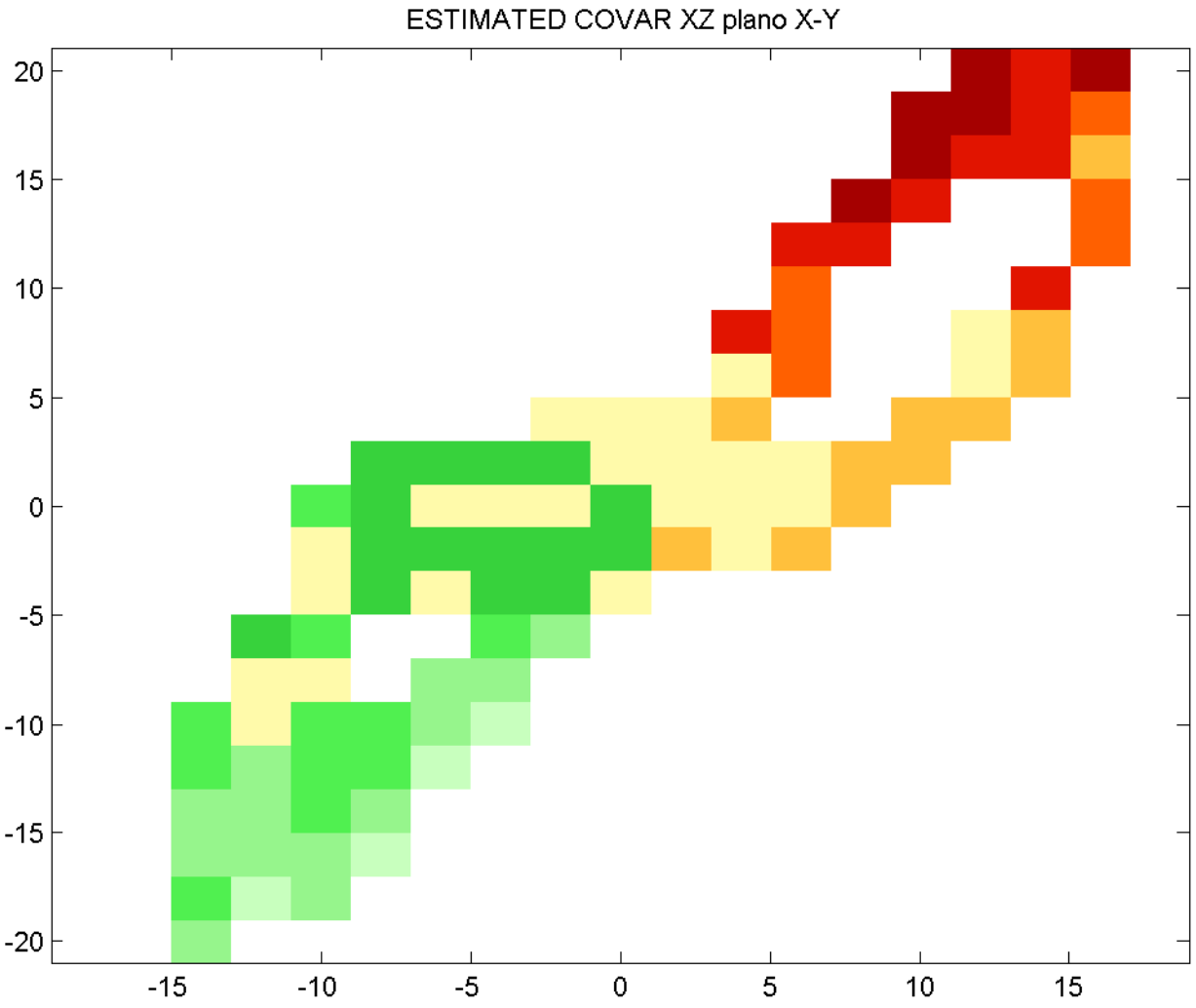
promedio.m : Computes the mean of a variable as a function of the position on the Lorenz atractor.

To run the code in type main_drive in Matlab under ./src

Example of X error variance as a function of X and Y

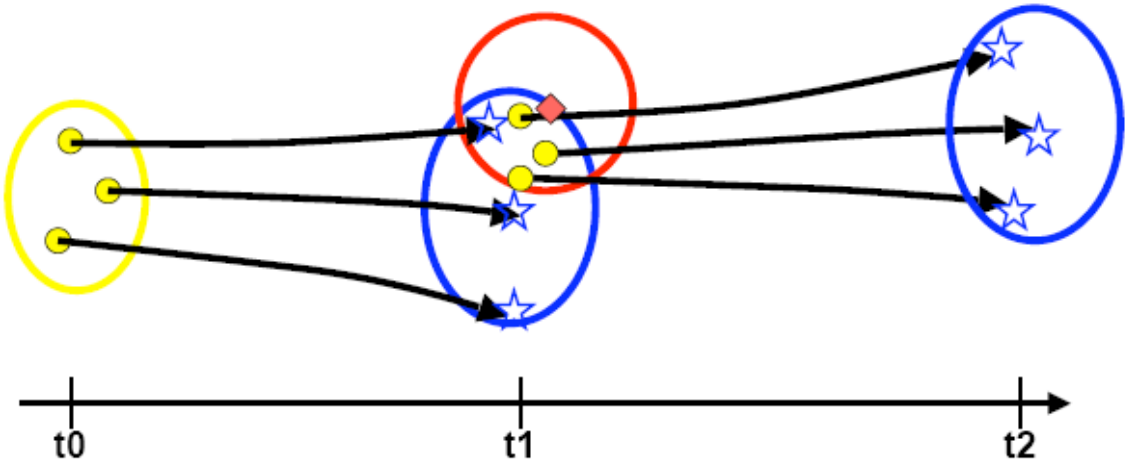


Example of XZ error covariance as a function of X and Y



ETKF scheme

Local Ensemble Transform Kalman Filter
(LETKF, Ott et al., 2004, Hunt et al., 2007)



✓ Yellow: analysis ensemble and its uncertainty; Blue: background ensemble and its uncertainty; Red: observation and its uncertainty

✓ LETKF, like any other EnKF, provides background and analysis uncertainty estimation in every analysis cycle.

Liu (2007)

ETKF equations: Hunt et. al. 2007

$$\bar{\mathbf{x}}^a = \bar{\mathbf{x}}^b + \mathbf{X}^b \bar{\mathbf{w}}^a$$

The analysis is a weighed average of the background ensemble members.

$$\mathbf{W}^a = [(k-1)\tilde{\mathbf{P}}^a]^{1/2} \qquad \mathbf{X}^a = \mathbf{X}^b \mathbf{W}^a$$

X^a, X^b and W^a are n*k matrices where n is the number of model variables.

k is the number of ensemble members.

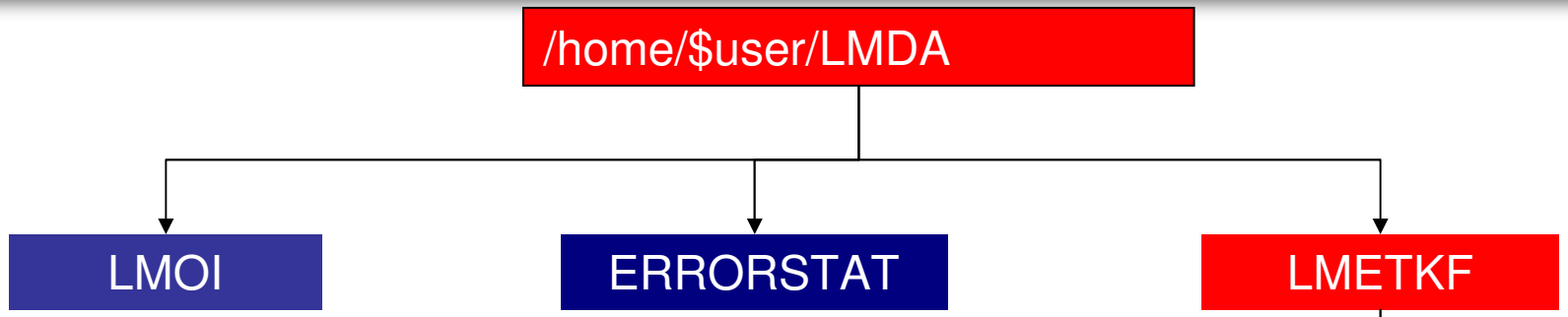
$$\bar{\mathbf{w}}^a = \tilde{\mathbf{P}}^a (\mathbf{Y}^b)^T \mathbf{R}^{-1} (\mathbf{y}^o - \bar{\mathbf{y}}^b),$$

$$\tilde{\mathbf{P}}^a = [(k-1)\mathbf{I} + (\mathbf{Y}^b)^T \mathbf{R}^{-1} \mathbf{Y}^b]^{-1}.$$

$$\mathbf{X}^b = \left[\mathbf{x}_1^b - \bar{\mathbf{x}}^b \,|\, \ldots \,|\, \mathbf{x}_K^b - \bar{\mathbf{x}}^b\right];$$

$$\mathbf{y}_i^b = H(\mathbf{x}_i^b); \; \mathbf{Y}_n^b = \left[\mathbf{y}_1^b - \bar{\mathbf{y}}^b \,|\, \ldots \,|\, \mathbf{y}_K^b - \bar{\mathbf{y}}^b\right]$$

Package



./src : matlab codes for model, data assimilation and plotting.

./output

Output states (truth, analysis, observations and background), plots

Matlab codes (IV)

- main_drive.m: **main driver**

  Parameters control model and data assimilation:

  1. bst: Observation/analysis interval (default=8)
  2. $\mathbf{R}_0$: Observation error variance
  3. iobs: observation locations (default [1;2;3], observing all variables)
  4. K: ensemble size
  5. e(1) and e(2) parameters for multiplicative and additive covariance inflation.

To run the code in type main_drive in Matlab under ./src

Matlab codes (V)

- Model

  L63eqs.m: governing equations

  stepit.m: forward integration by Runge-Kutta method

- DA procedure

  DA_init.m: initialize matrix operators (**B**, **R**, and **H**) for OI

  RUN_ETKF.m: compute ETKF analysis

- Plotting tools

  Daerrplt.m: plot the RMS analysis error

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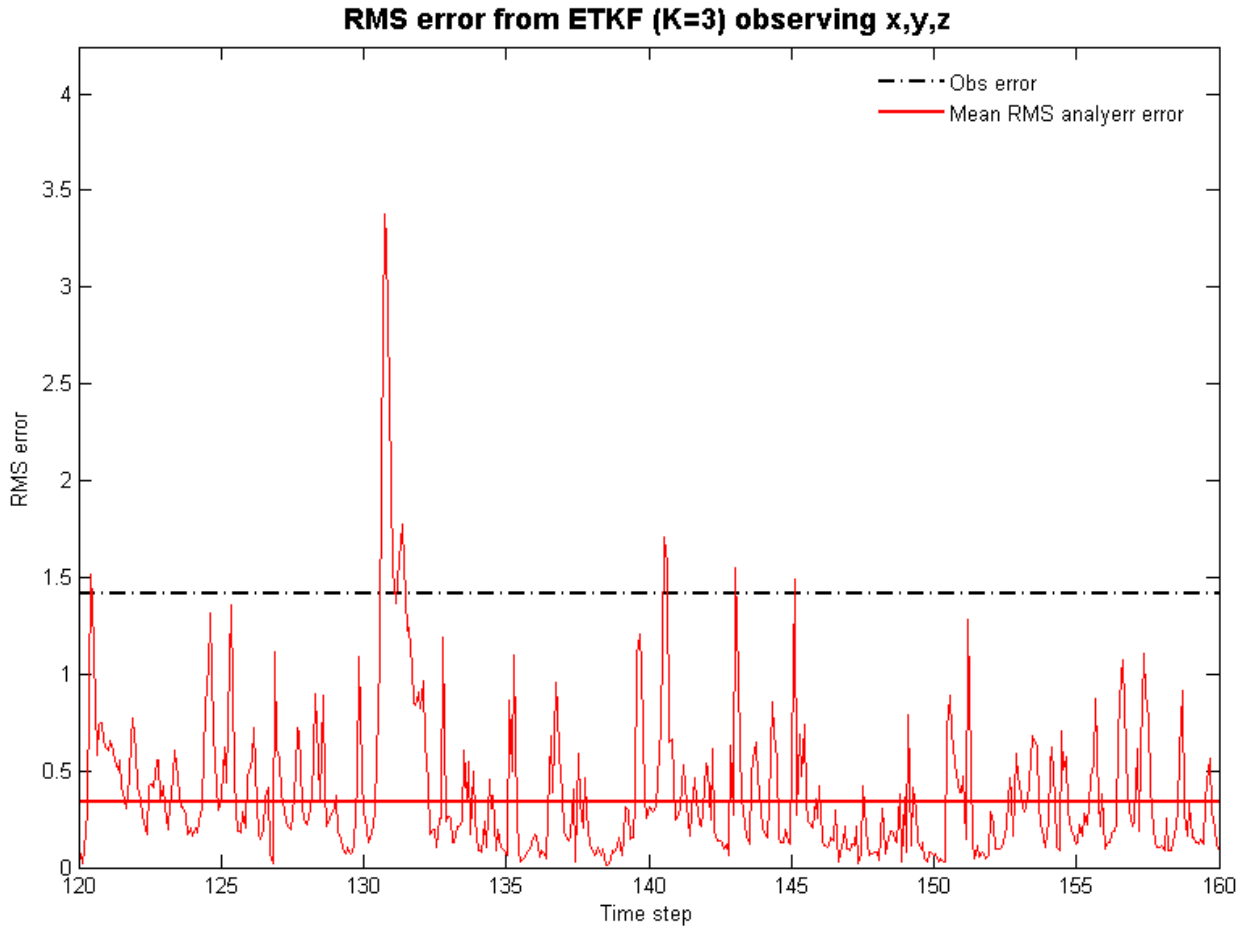
Example of ETKF analysis error

(observing x,y,z every 8 time-step)



Example: analysis vs. truth

for x variable (observing x,y,z every 8 time-step)