ECMWF Data Assimilation

Contribution from many people
Fundamental challenges of ECMWF 4D-Var Assimilation System

- Incremental 4D-Var
- Linearized Physics
- Observation handling
- Assimilation of satellite data
- Model and Observation Bias
- Modelling Background Covariance
The observations are used to correct errors in the short forecast from the previous analysis time.
Every 12 hours we assimilate 4 – 8,000,000 observations to correct the 100,000,000 variables that define the model’s virtual atmosphere.
This is done by a careful 4-dimensional interpolation in space and time of the available observations; this operation takes as much computer power as the 10-day forecast.
All observations within a 12-hour period are used simultaneously in one global (iterative) estimation problem.

- Observation minus model differences are computed at the observation time using the full forecast model at T799 (25km) resolution.
- 4D-Var finds the 12-hour forecast evolution that optimally fits the available observations. A linearized forecast model is used in the minimization process based on the adjoint method.
- It does so by adjusting surface pressure, the upper-air fields of temperature, wind, specific humidity and ozone.
- The analysis vector consists of \(\sim 10^8\) elements at T255 resolution (80km).
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4D Variational Data Assimilation

- Model: $\mathcal{M}$,
- Observations: $y$,
- Background: $x_b$,
- Observation operator: $\mathcal{H}$,
- Cost function to minimise:
  \[ J(x) = \frac{1}{2}(x-x_b)^T B^{-1} (x-x_b) + \frac{1}{2} [y - \mathcal{H}(x)]^T R^{-1} [y - \mathcal{H}(x)] + J_c \]

- In discrete form:
  \[ J(x) = \frac{1}{2} (x-x_b)^T B^{-1} (x-x_b) + \frac{1}{2} \sum_{i=0}^{n} [y_i - \mathcal{H}_i(\mathcal{M}_i(x))]^T R_i^{-1} [y_i - \mathcal{H}_i(\mathcal{M}_i(x))] \]
Incremental 4D-Var

- The cost function is expressed in terms of increments with respect to the background state \( \delta x = x - x_b \).

- \( \mathcal{H} \) and \( \mathcal{M} \) are linearised around \( x_i = \mathcal{M}_i(x_0) \).

\[
J(\delta x) = \frac{1}{2} \delta x^T B^{-1} \delta x + \frac{1}{2} \sum_{i=0}^{n} (H_i M_i \delta x - d_i)^T R_i^{-1} (H_i M_i \delta x - d_i)
\]

where

- \( H_i \) and \( M_i \) are the linearised observation operator and model,
- \( d_i = y_i - \mathcal{H}_i(\mathcal{M}_i(x_b)) \) are the innovations.

- The innovations, which are the primary input to the assimilation, are always computed using the full observation operator and model to ensure the highest possible accuracy.
4D-Var incremental formulation

Courtier, Thépaut and Hollingsworth (1994)

In the incremental formulation the cost function $J$ is expressed in terms of increments $\delta x = x - x_b$ with respect to the background state at initial time. $H_i$ and $M_i$ are the TL of $\mathcal{H}_i$ and $\mathcal{M}_i$ linearized around $x(t_i) = \mathcal{M}_i(x_b(t_0))$.

$$J(\delta x) = \delta x^T B^{-1} \delta x + \sum_{i=1}^{N} (H_i M_i \delta x - d_i)^T R^{-1} (H_i M_i \delta x - d_i)$$

The $i$-summation is over $N=25$ ½-hour long time slots of the 12-hour assimilation period.

The innovations $d_i$ are calculated using the non-linear operators, $\mathcal{H}_i$ and $\mathcal{M}_i$.

$$d_i = y_i - \mathcal{H}_i(\mathcal{M}_i(x_b(t_0)))$$

This ensures the highest possible accuracy for the calculation of the innovations $d_i$ which are the primary input to the assimilation!
The Outer Iterations

After each minimisation at inner level:

- $x$ is updated: $x_a^j = x_b + \delta x_j$,

- $H_i$ and $M_i$ are re-linearised around $x_i = M_i(x_a^j)$,

- Innovations are re-calculated using the full nonlinear observation operator $H$ and model $M$:
  \[ d_i^j = y_i - H_i(M_i(x_a^j)) \]

- Superscript $j$ represents the outer iteration.

- The nonlinear model $M$ remains at T799 throughout.
The Inner Iterations

- Tangent Linear approximation:

\[ \mathcal{M}(x + \delta x) \approx \mathcal{M}(x) + M\delta x \quad \text{and} \quad \mathcal{H}(x + \delta x) \approx \mathcal{H}(x) + H\delta x \]

- Approximations to reduce cost: the tangent linear model (and its adjoint) is degraded with respect to the full model \( \mathcal{M} \):
  - Lower resolution \( \text{T255 instead of T799} \)
  - Simplified physics (some processes are ignored),
  - Simpler dynamics (spectral instead of grid-point humidity).

- This results in shorter control vector and cheaper TL and AD models during the minimisation.
**ECMWF 4D-Var procedure**

Use all data in a 12-hour window (0900-2100 UTC for 1200 UTC analysis)

1. Group observations into ½ hour time slots
2. Run the T799 (25km) high resolution forecast from the previous analysis and compute “observation”- “model” differences

3. Adjust the model fields at the start of assimilation window (0900 UTC) so the 12-hour forecast better fits the observations. This is an iterative process using a lower resolution linearized model T95 (210km), T159 (125km) or T255 (80 km) and its adjoint model

4. Rerun the T799 high resolution model from the modified (improved) initial state and calculate new observation departures

5. The 3-4 loop is repeated three times to produce a good high resolution estimate of the atmospheric state – the result is the ECMWF analysis
Multi-incremental quadratic 4D-Var at ECMWF
Outer loop forecast resolution is now T799
Important for accurate comparison against observations

T511 (40km)

10m winds

T799 (25km)
Since February 2006
91-level vertical resolution from Feb 2006

Fit of L91 and L60 background and analysis to NH radiosonde T

Pressure (hPa)

Level number

Pressure

K

0 1 2 3
Katrina: 90h forecasts valid 18UTC 29 Aug
T511 (40km) versus T799 (25km):

Model resolution is important, especially for extreme events.
4D-Var with three inner loop: efficient, accurate and allows non-linearity

- M.Level 80 (900 hPa) temperature analysis increments for each of the three minimizations.
- Decreasing amplitudes $T_{95} > T_{159} > T_{255}$.
- Small corrections added at $T_{255}$ where data density is highest.
- Model and observ. operators are re-linearized twice.

Add $T_{95}$ increment to $T_{799}$ BG and re-linearize $M, H$

Add $T_{159}$ increment and re-linearize $M, H$

Add $T_{255}$ increment = final $T_{799}$ analysis
Fundamental challenges of ECMWF
4D-Var Assimilation System

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ECMWF LINEARIZED PHYSICS

OPERATIONAL VERSION

Dynamics
\( u, v, T, q \)

Next time step

Radiation
SW+reduced LW

Vertical diffusion

Gravity wave drag

Cloud & condensation

Convection

new moist parametrization used since 2007
Zonal wind increments at model level ~ 1000 hPa [24-hour integration]

FD = Finite Difference

TL_{ADIAB} = adiabatic TL model

TL_{WSPHYS} = TL model with the whole set of simplified physics
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Conventional observations used

**SYNOP/METAR/SHIP:**
- MSL Pressure
- 10m-wind
- 2m-Rel.Hum.

**DRIBU:**
- MSL Pressure
- Wind-10m

**Radiosonde balloons (TEMP):**
- Wind
- Temperature
- Spec. Humidity

**PILOT/Profilers:**
- Wind

**Aircraft:**
- Wind
- Temperature

Note: We only use a limited number of the observed variables; especially over land.
Satellite data sources used in the operational ECMWF analysis

- 13 Sounders: NOAA AMSU-A/B, HIRS, AIRS, IASI, MHS
- 5 imagers: 3xSSM/I, AMSR-E, TMI
- 3 Scatterometer sea winds: ERS, ASCAT, QuikSCAT
- Geostationary, 4 IR and 5 winds
- 2 Polar, winds: MODIS
- 4 ozone
- 6 GPS radio occultation
Data extraction
- Check out duplicate reports
- Ship tracks check
- Hydrostatic check

Thinning
- Data not used to avoid over-sampling and correlated errors
- Even so departures from background and analysis are generated and usage flags also

Blacklisting
- Data skipped due to systematic bad performance or due to different considerations (e.g. data being assessed in passive mode)
- Departures and flags available for further assessment

Model/4D-Var dependent QC
- First guess based rejections
- VarQC rejections

Used data → Increments

Analysis
### Observation data count for one 12h 4D-Var cycle

**0900-2100UTC 3 March 2008**

<table>
<thead>
<tr>
<th>Screened</th>
<th>Assimilated</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Synop:</strong></td>
<td><strong>Synop:</strong></td>
</tr>
<tr>
<td>450,000</td>
<td>64,000</td>
</tr>
<tr>
<td>0.3%</td>
<td>0.7%</td>
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<tr>
<td><strong>Aircraft:</strong></td>
<td><strong>Aircraft:</strong></td>
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<tr>
<td>434,000</td>
<td>215,000</td>
</tr>
<tr>
<td>0.3%</td>
<td>2.4%</td>
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<tr>
<td><strong>Dribu:</strong></td>
<td><strong>Dribu:</strong></td>
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<tr>
<td>24,000</td>
<td>7,000</td>
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<tr>
<td>0.02%</td>
<td>0.1%</td>
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</tr>
<tr>
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<td>76,000</td>
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<tr>
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<td>0.8%</td>
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<tr>
<td><strong>Pilot:</strong></td>
<td><strong>Pilot:</strong></td>
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<tr>
<td>86,000</td>
<td>39,000</td>
</tr>
<tr>
<td>0.1%</td>
<td>0.4%</td>
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<tr>
<td><strong>AMV’s:</strong></td>
<td><strong>AMV’s:</strong></td>
</tr>
<tr>
<td>2,535,000</td>
<td>125,000</td>
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<tr>
<td>1.6%</td>
<td>1.4%</td>
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<td><strong>Radiance data:</strong></td>
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<td>8,207,000</td>
</tr>
<tr>
<td>96.9%</td>
<td>91.0%</td>
</tr>
<tr>
<td><strong>Scat:</strong></td>
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</tr>
<tr>
<td>835,000</td>
<td>149,000</td>
</tr>
<tr>
<td>0.5%</td>
<td>1.7%</td>
</tr>
<tr>
<td><strong>GPS radio occult:</strong></td>
<td><strong>GPS radio occult.:</strong></td>
</tr>
<tr>
<td>271,000</td>
<td>137,000</td>
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<tr>
<td>0.2%</td>
<td>1.5%</td>
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<tr>
<td><strong>TOTAL:</strong></td>
<td><strong>TOTAL:</strong></td>
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<tr>
<td>155,448,000</td>
<td>9,018,000</td>
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<tr>
<td>100.00%</td>
<td>100.00%</td>
</tr>
</tbody>
</table>

**99% of screened data is from satellites**

**96% of assimilated data is from satellites**
Quality control: Good example

In the quality control procedure all data from different data types are simultaneously checked. In this Australian example the presence of AIRCRAFT data has led to the rejection of a PILOT wind.
Quality control: Bad example
Tropical Cyclone

Observations of intense and small-scale features may be rejected although the measurements are correct.

The problem occurs when the resolution of the analysis system (as determined by the B-matrix) is insufficient.
Quality control: Bad example
Extreme event -
The ‘French Storm’, 27 Dec 1999

Observations of intense and small-scale features may be rejected although the measurements are correct.
The Huber-norm –

Gaussian
Huber
Gaussian + flat
Comparing optimal observation weights
Huber-norm (red) vs. Gaussian+flat (blue)

North Hemisphere Temperature from Radiosonde
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Combined impact of all satellite data

Anomaly correlation geopotential height 500 hPa

Southern hemisphere: ~3 days at day 5

Northern hemisphere: ~2/3 to 3/4 of a day at day 5
Assimilation of rain-affected microwave radiances

- Assimilation of rain-affected SSM/I radiances in 1D+4D-Var active since June 2005.
- Main difficulties: inaccurate moist physics parameterizations (location/intensity), formulation of observation errors, bias correction, linearity.
- Major improvements accomplished in 2007 and SSMIS, TMI, AMSR-E data included.
- Direct 4D-Var radiance assimilation envisaged for 2008.
Forecast sensitivity to observations in analysis

CY32R1, T511L60, 20070105–20070212

SSM/I clear-sky, winter

SSM/I clouds/rain, winter

Mean 36–12h precipitation forecast initialized at 12 UTC

[J/kg]

[10^{-3} \text{ mm}]
Advanced infrared sounders: AIRS and IASI

**AIRS**
- Operational at ECMWF since October 2003.
- 324 channels received.
- Up to 155 channels may be assimilated (CO$_2$ and H$_2$O bands).

**IASI**
- Operational at ECMWF since June 2007.
- 8461 channels received
- 366 channels routinely monitored.
- Up to 168 channels may be assimilated (CO$_2$ band only).

500 hPa geopotential anomaly correlation
(56 cases, spring 2007, normalized RMSE difference, own analysis)
Assimilation of GPS radio occultation data

- COSMIC bending angles assimilated (both rising and setting occultations), CHAMP, GRACE as back-up, GRAS being monitored.
- GPSRO data reveals a warm bias of aircraft observations.
- Substantial improvement of stratospheric T-biases.
- METOP GRAS data assimilated since May 2008.
- COSMIC-2 follow-on?

Southern Hemisphere scores
(normalized AC differences)

Improved fit to radiosonde data
Recent revisions to observation usage

- Use AMSU-B on NOAA-16, 17 and 18, AMSU-A on NOAA-18 and 20
- Operational assimilation of 155 EOS-1/AIRS channels
- Operational assimilation of 168 METOP/IASI channels
- Radiance from Meteosat-5 (India), GOES-9 (Japan) and 12 (replaces GOES-8) and Meteosat-8
- Atmosphere feature track. winds from GOES-12, Meteosat-8, MTSAT-1R
- Use MODIS winds from AQUA satellite
- Use of SCIAMACHY and more SBUV ozone retrievals
- METAR surface pressure data active
- Increase use of radiosonde humidity in upper troposphere
- Use GPS radio occultation data (COSMIC)
Recent revisions to the assimilation system

- Increased resolution from T511/T159 L60 to T799/T255 L91
- Now three outer loop: T799 outer with T95/T159/T255 inner loop
- Use grid-point humidity and ozone in 4D-Var analysis
- More advanced Tangent Linear physics scheme in 4D-Var
- Wavelet Jb formulation has been implemented
- Jb statistics from latest ensemble data assimilation
- New humidity analysis using a normalized control variable
- Assimilation of “Cloudy and rainy radiances”
- Variational bias correction of satellite radiances
- Adaptive bias correction for radiosondes and SYNOP pressure data
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Model bias:
upper-stratospheric model errors variation

T255L60 model currently used for ERA-Interim

Mean temperature [K] 120-hour forecast errors for experiment 1112: Arctic

Summer: Radiation, ozone?

Mean temperature [K] 120-hour forecast errors for experiment 1112: Antarctica

Winter: Gravity-wave drag?

0.1hPa (65km)

20hPa (22km)
Observation bias:
Radiosonde temperature observations

Daytime warm bias due to radiative heating of the temperature sensor (depends on solar elevation and equipment type)

Mean temperature anomalies for different solar elevations

Bias changes due to change of equipment

Observed – ERA-40 background at Saigon (200 hPa, 0 UTC)
Observation bias:
Satellite radiances

Monitoring the background departures (averaged in time and/or space):

- **Constant bias (HIRS channel 5)**: A graph showing a constant bias over time.

- **Diurnal bias variation in a geostationary satellite**: A graph depicting diurnal variations in temperature over the months of September to November.

- **Air-mass dependent bias (AMSU-A channel 14)**: A map showing the bias depending on scan position across different latitudes and longitudes.
Implications for data assimilation:  
The effect of model bias on trend estimates

Most assimilation systems assume unbiased models and unbiased data

Biases in models and/or data can induce spurious trends in the assimilation
Variational bias correction: The modified analysis problem

The original problem:

\[ J_{\text{b}}: \text{background constraint} \]

\[ J(x) = (x_b - x)^T B^{-1}_x (x_b - x) + [y - h(x)]^T R^{-1}_y [y - h(x)] \]

The modified problem:

\[ J_{\text{bo}}: \text{observation constraint} \]

\[ J_{\beta_{\text{b}}}: \text{background constraint for } \beta \]

\[ J(x, \beta) = (x_b - x)^T B^{-1}_x (x_b - x) + (\beta_b - \beta)^T B^{-1}_\beta (\beta_b - \beta) + [y - b_o(x, \beta) - h(x)]^T R^{-1}_y [y - b_o(x, \beta) - h(x)] \]

Parameter estimate from previous analysis

\[ J_{\text{bo}}: \text{bias-corrected observation constraint} \]

Dee 2004, 2005
Performance:

Adaptive bias correction of NOAA-17 HIRS Ch12

- \( p(0) \): global constant
- \( p(1) \): 1000-300hPa thickness
- \( p(2) \): 200-50hPa thickness
- \( p(3) \): surface temperature
- \( p(4) \): total column water
Performance:

NOAA-9 MSU channel 3 bias corrections (cosmic storm)

200 hPa temperature departures from radiosonde observations
Weak-constraint 4D-Var

Include model error in the control vector:

\[
J(x_0, \eta) = \frac{1}{2} \sum_{i=0}^{n} [\mathcal{H}(x_i) - y_i]^T R_i^{-1} [\mathcal{H}(x_i) - y_i] \\
+ \frac{1}{2} (x_0 - x_b)^T B^{-1} (x_0 - x_b) + \eta^T Q^{-1} \eta
\]

with \( x_i = M_i(x_{i-1}) + \eta_i \).

Constraint is determined by \( Q \)

Tremolet 2007
Lindskog 2008
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Importance of Background Covariances

• The formulation of the $J_b$ term of the cost function is *crucial* to the performance of current analysis systems.

• To see why, suppose we have a single observation of the value of a model field at one gridpoint.

• For this simple case, the observation operator is:

$$H = (0,...,0,1,0,...,0).$$

• The gradient of the 3dVar cost function is:

$$\nabla J = B^{-1}(x-x_b) + H^TR^{-1}(Hx-y) = 0$$

• Multiply through by $B$ and rearrange a bit:

$$x - x_b = B H^T R^{-1}(y-Hx)$$

• But, for this simple case, $R^{-1}(y-Hx)$ is a scalar
Importance of Background Covariances

- So, we have: \( \mathbf{x} - \mathbf{x}_b \propto \mathbf{B}\mathbf{H}^T \)

- But, \( \mathbf{H} = (0, \ldots, 0, 1, 0, \ldots, 0) \)

- \( \Rightarrow \) **The analysis increment is proportional to a column of \( \mathbf{B} \).**

- The role of \( \mathbf{B} \) is:
  1. To spread out the information from the observations.
  2. To provide statistically consistent increments at the neighbouring gridpoints and levels of the model.
  3. To ensure that observations of one model variable (e.g. temperature) produce dynamically consistent increments in the other model variables (e.g. vorticity and divergence).
Estimating Background Error Statistics from Ensembles of Analyses

Normal Analysis

\[ x^b \]
\[ y \]
SST (etc.)

\[ x^a \]

Analysis

\[ x^f \]

Forecast

Perturbed Analysis

\[ X^{t+\varepsilon_{\text{Stochastics}}} \]
\[ y^{+\varepsilon^o} \]
SST\(+\varepsilon^{\text{SST}}\) (etc.)

\[ x^{a+\varepsilon^a} \]

Analysis

\[ x^{f+\varepsilon^f} \]

Forecast
The ECMWF $J_b$ Formulation – the balance operator

- The most obvious correlation in the background errors is the balance between mass errors and wind errors in the extra-tropics.
- We therefore define our change of variable as:
  \[ L = K B_u^{1/2} \]
- where $K$ accounts for all the correlation between variables (e.g. between the mass and wind fields).
- The matrix $B_u$ is a covariance matrix for variables that are uncorrelated with each other.
The ECMWF $J_b$ Formulation – The balance operator

\[
\begin{pmatrix}
\zeta \\
D \\
(T, p_s) \\
q
\end{pmatrix} = \begin{pmatrix}
I & 0 & 0 & 0 \\
M & I & 0 & 0 \\
N & P & I & 0 \\
0 & 0 & 0 & I
\end{pmatrix} \begin{pmatrix}
\zeta \\
D_u \\
(T, p_s)_u \\
q
\end{pmatrix}
\]

- The most important part of the balance operator is the sub-matrix N, which calculates a balanced part of $(T, p_s)$, determined from the vorticity.
- N is implemented in 2 parts:
  1. A balanced “geopotential” is calculated from $\zeta$.
  2. Balanced $(T, p_s)$ are calculated using statistical regression between $(T, p_s)$ and geopotential.
Increments from a single observation

Increments due to a single observation of geopotential height at 1000 hPa 60N with value 10m below background
Increments for a single observation of geopotential height at 1000hPa
NSCAT wind information vertically propagated and impact on other variables

- No SCAT temperature increments 1996102600 (awlj)
- No SCAT wind speed increments 1996102600 (awlj)
- NSCAT temperature increments 1996102600 (axuk)
- NSCAT wind speed increments 1996102600 (axuk)
Ensembles of data assimilations

- Run an ensemble of analyses with randomly perturbed observations, perturbed SST field and stochastic physics perturbations.
- Compute the variance of the short-range forecast fields.
- The spread will have the statistical characteristics of short-range forecast error and it will be flow dependent.

**850 hPa U-Comp 1-Month EnDA Mean Spread of 3-hour forecast**
Reanalysis

- ERA–Interim is current ECMWF reanalysis project following ERA–15 & 40.

The ERA-40 observing system:

- ERA-40 observations until August 2002
- ECMWF operational data after August 2002
- Reprocessed altimeter wave-height data from ERS
- Humidity information from SSM/I rain-affected radiance data
- Reprocessed METEOSAT AMV wind data
- Reprocessed ozone profiles from GOME
- Reprocessed GPSRO data from CHAMP

ERA-Interim

- 1989