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Balance and 4D-Var

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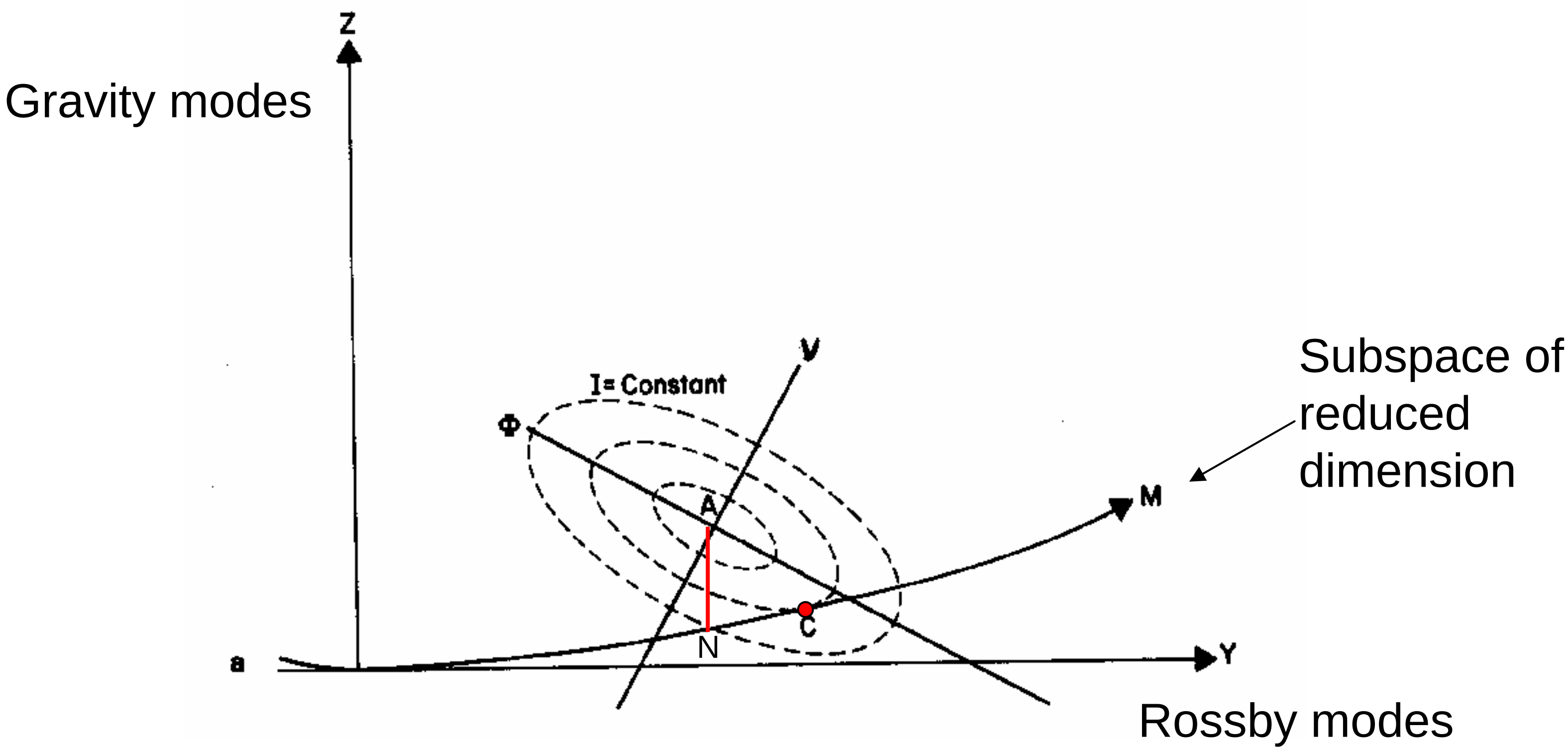
Topics

- Combining analysis and initialization steps
- Choosing a control variable
- Impacts of imbalance
  - Filtering of GWs in troposphere impacts mesopause temperatures and tides
  - Noisy wind analyses impact tracer transport

Combining Analysis and Initialization steps

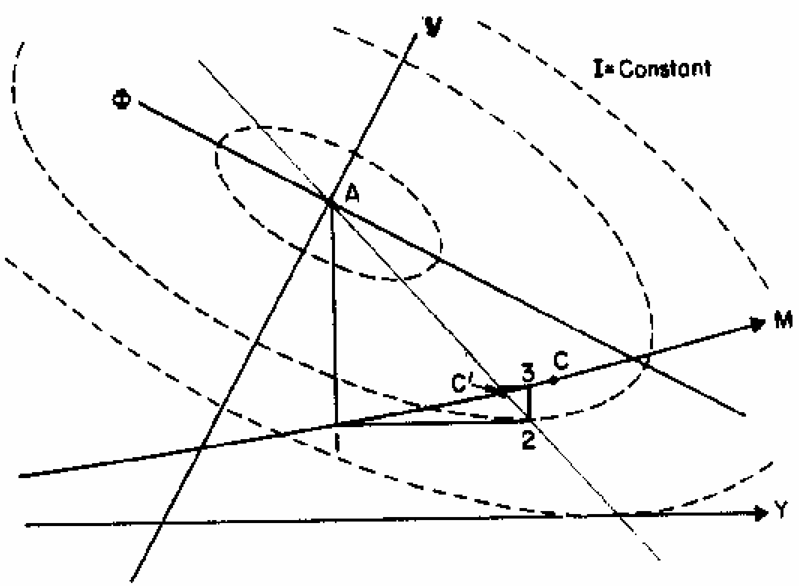
- Doing an analysis brings you close to the data.
- Doing an initialization moves you farther from the data.



Daley (1986)

Variational Normal Mode Initialization

Daley (1978), Tribbia (1982), Fillion and Temperton (1989), etc.



Daley (1986)

1. NNMI from A to 1
2. Minimize distance to A holding G fixed (1 to 2)
3. NNMI from 2 to 3
4. Minimize distance to A holding G fixed (3 to C')

4D-Var with strong constraints

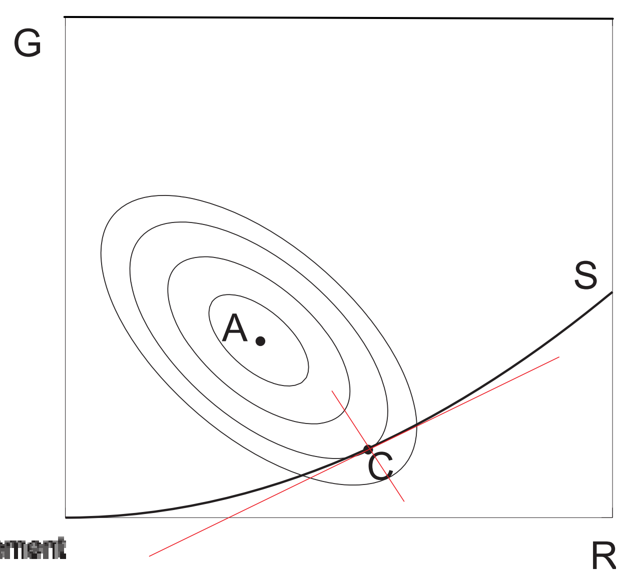
Minimize $J(\mathbf{x}_0)$ subject to the constraints: $\hat{c}_i(\mathbf{x}_0) = 0, \quad i = 1,\ldots,t.$

Necessary and sufficient conditions for $\mathbf{x}_0$ to be a minimum are:

1. $\hat{c}(\tilde{\mathbf{x}}_0) = 0$
2. $\mathbf{Z}(\tilde{\mathbf{x}}_0)^T \nabla J(\tilde{\mathbf{x}}_0) = 0$
3. $\mathbf{Z}(\tilde{\mathbf{x}}_0)^T \left[\mathbf{H} - \sum_{i=1}^{t} \lambda_i \hat{\mathbf{G}}_i(\tilde{\mathbf{x}}_0) \right] \mathbf{Z}(\tilde{\mathbf{x}}_0)$ is pos. def.

Projection onto constraint tangent

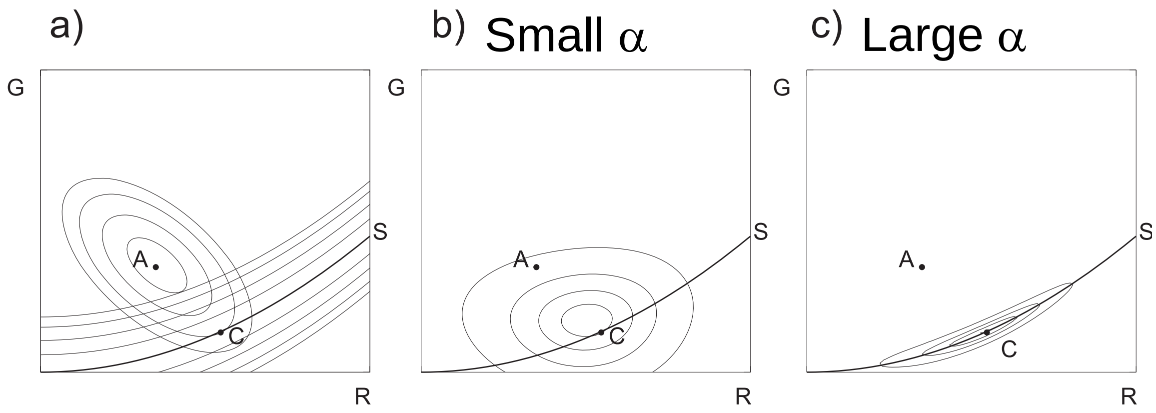
Hessian of constraints



Gill, Murray, Wright (1981)

4D-Var with weak constraints

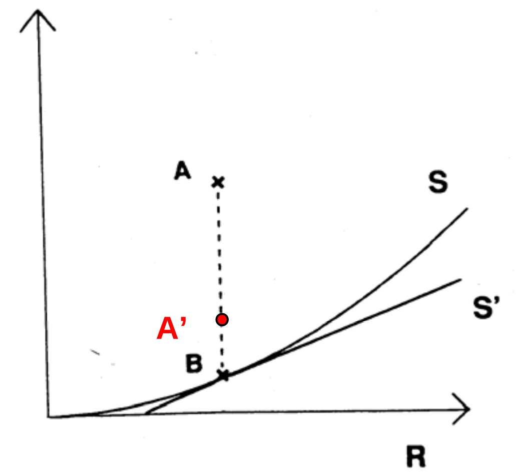
Penalty Methods: Minimize $J_{weak} = J_{4DVAR} + \frac{\alpha}{2}\hat{c}(\mathbf{x})^{\mathrm{T}}\hat{c}(\mathbf{x})$



4D-Var with NNMI constraints

Strong constraint

…owing to the iterative and approximate character of the initialization algorithm, the condition || dG/dt || = 0 cannot in practice be enforced as an exact constraint.
Courtier and Talagrand (1990)



Weak constraint

$$J = J_{4DVAR} + \alpha \left\| \frac{d\hat{\mathbf{c}}_G}{dt} \right\|^2$$

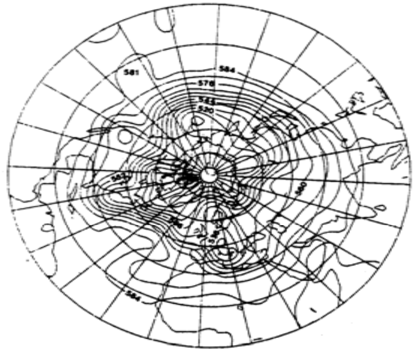


Fig. 3. 500 mb height field for 00:00 GMT, 19 March 1985, as produced by the operational assimilation and forecast EMERAUDE system of Direction de la Météorologie, Paris (unit:dam).

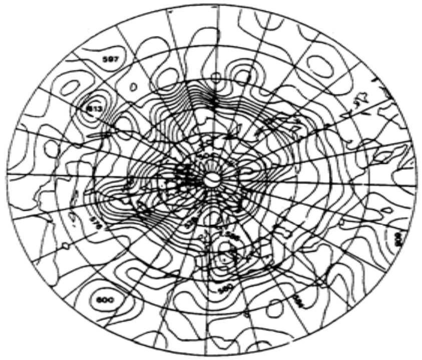


Fig. 4. Height field produced at the end of the assimilation period (00:00 GMT, 19 March 1985) by minimization of the distance function (5.1) (unit:dam).

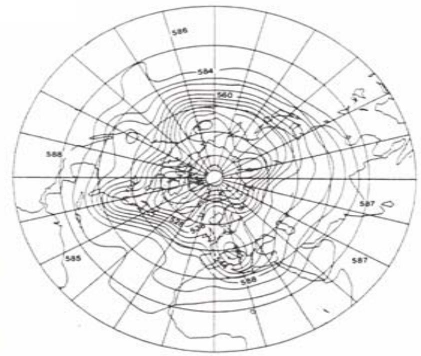


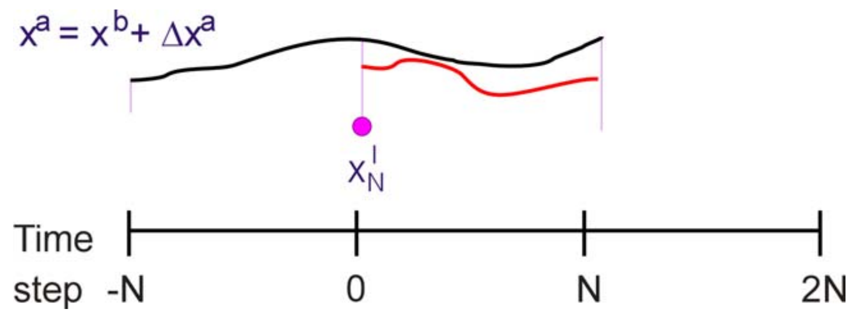
Fig. 8. As Fig. 4, but for a distance function penalized with term (5.3).

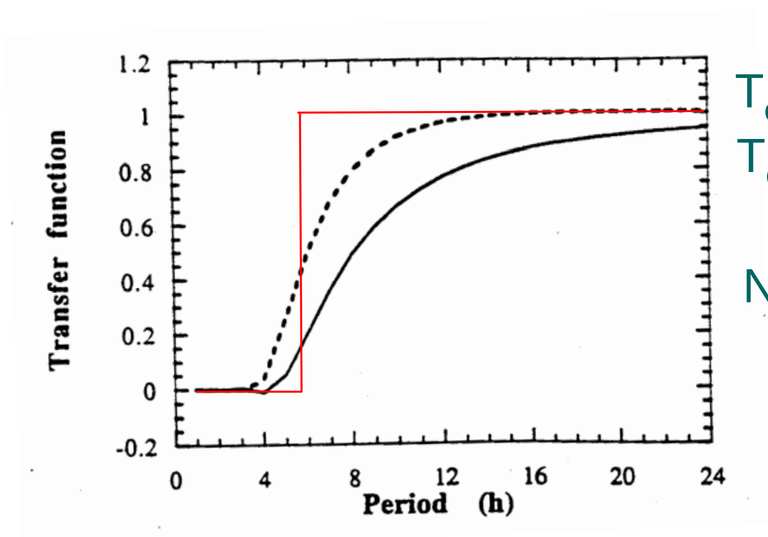
Courtier and Talagrand (1990)

Digital Filter Initialization

Lynch and Huang (1992)

$$x_0^I = \sum_{k=-N}^{N} h_k x_k^u$$





T_c=6 h

T_c=8 h

N=12, Δt=30 min

Fillion et al. (1995)

4D-Var with DFI constraints

Strong Constraint

- Because filter is not perfect, some inversion of intermediate scale noise occurs, but DFI as a strong constraint suppresses small scale noise. (Polavarapu et al. 2000)

Weak Constraint

- Introduced by Gustafsson (1993)
- Weak constraint can control small scale noise (Polavarapu et al. 2000)
- Implemented operationally at Météo-France (Gauthier and Thépaut 2001)

Recent approaches

- Early work on incorporating balance constraints in 4D-Var used a balance on the full state
- Since Courtier et al. (1994), most groups use an incremental approach so balance should be applied to analysis increments
- Even without penalty terms, background error covariances can be used to create balanced increments. So what is the best choice of analysis variables ?

Choice of control variable

$$J(\mathbf{x}_0)=\frac{1}{2}(\mathbf{x}_0-\mathbf{x}_b)^T\mathbf{B}^{-1}(\mathbf{x}_0-\mathbf{x}_b)+\frac{1}{2}\sum_{k=0}^{N}(\mathbf{z}_k-H(\mathbf{x}_k))^T\mathbf{R}^{-1}(\mathbf{z}_k-H(\mathbf{x}_k))$$

- If obs and representativeness errors are spatially uncorrelated, $\mathbf{R}$ is diagonal so $\mathbf{R}^{-1}$ is easy
- To avoid inverse computation of full matrix, $\mathbf{B}$, use a change of control variable, $L\chi=(\mathbf{x}_0-\mathbf{x}_b)$ for $\mathbf{B}=LL^T$. Then $J(\chi)=\frac{1}{2}\chi^T\chi+J_{obs}$.
  - Note L is not stored as a matrix but a sequence of operations
  - If no obs, this is a perfect preconditioner since Hessian = I
- L can include a change of variable. Is there a natural, physical choice for control variable?

Choice of control variable - 2

- Balanced and unbalanced variables are uncorrelated (Daley 1991)
- Consider change of control variable, $\mathbf{x}=K\mathbf{x}^u$
- Then $\mathbf{B}=K\mathbf{B}^uK^T$. Background error covariances are defined in terms of uncorrelated variables.
- Original 3D-Var implementation of Parrish and Derber (1992) used geostrophic departure and divergence (both unbalanced) and vorticity (balanced) (Φ^u, D^u, ζ)

NNMI and balance constraints

Leith (1980) f-plane, Boussinesq (small vert scales)

	Order	0	1	2
Physical space	Mass-wind	Geostrophy $\nabla^2\Phi = f\nabla^2\Psi$	Nonlinear balance $\nabla^2\Phi = f\nabla^2\Psi + 2(\Psi_{xx}\Psi_{yy} - (\Psi_{xy})^2)$	
	Divergence or vert velocity	Non Divergence	QG omega eq.	
Normal mode approach	Gravity modes	$G(t=0)=0$	$\frac{dG}{dt}(t=0)=0$	$\frac{d^2G}{dt^2}(t=0)=0$
	Rotational modes	Nondivergent geostrophic	Nonlinear balance, QG omega eq.	

Parrish and Derber (1992) use div, geostr imbalance as control variables

Fisher (2003) uses departure from nonlinear balance, QG omega eq. for control variables

Parrish (1988), Heckley et al. (1992) use normal mode amp. as control variables

Fillion et al. (2007), Kleist et al. (2008) use INMI as strong constraint on increments

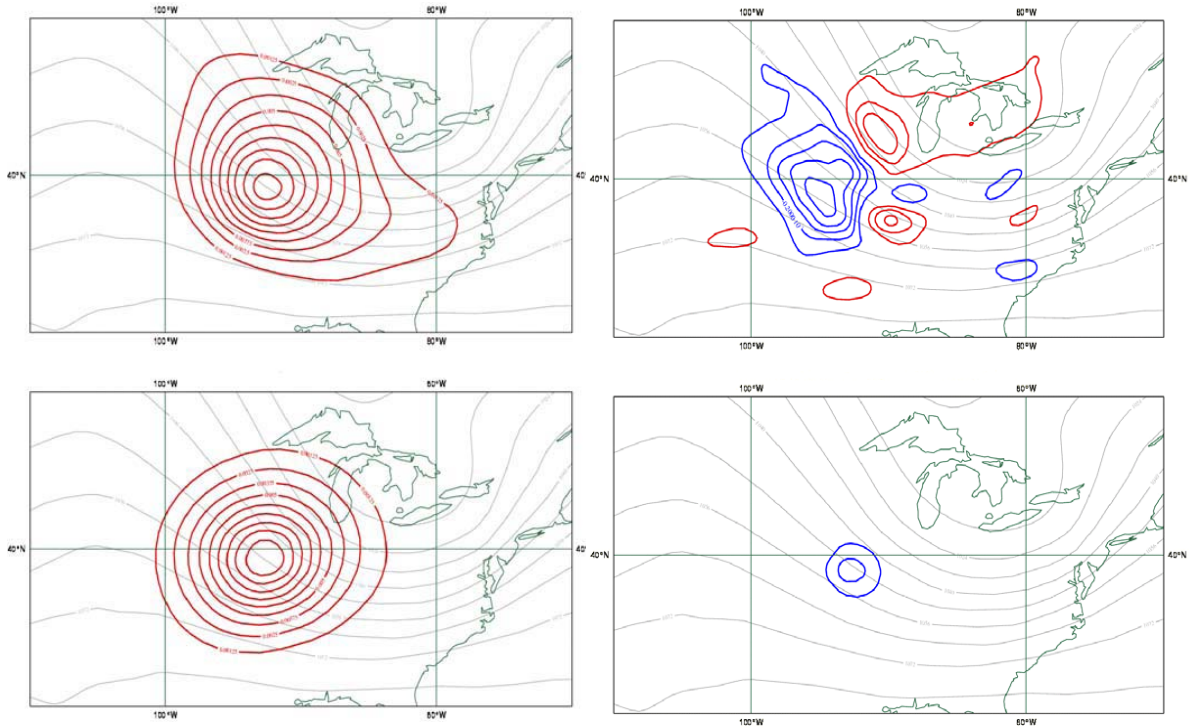
Nonlinear balanced variables require linearization around background state and introduce flow dependence of increments

Analysis increments from single Φ obs at 300 hPa with 4D-Var

ΔT 228 hPa | ΔD 228 hPa

Nonlinear balance

Statistical balance



grey Φ_{250}

Impact expected where curvature of background flow is high or in dynamically active regions

Fisher (2003, ECMWF)

The balanced control variable

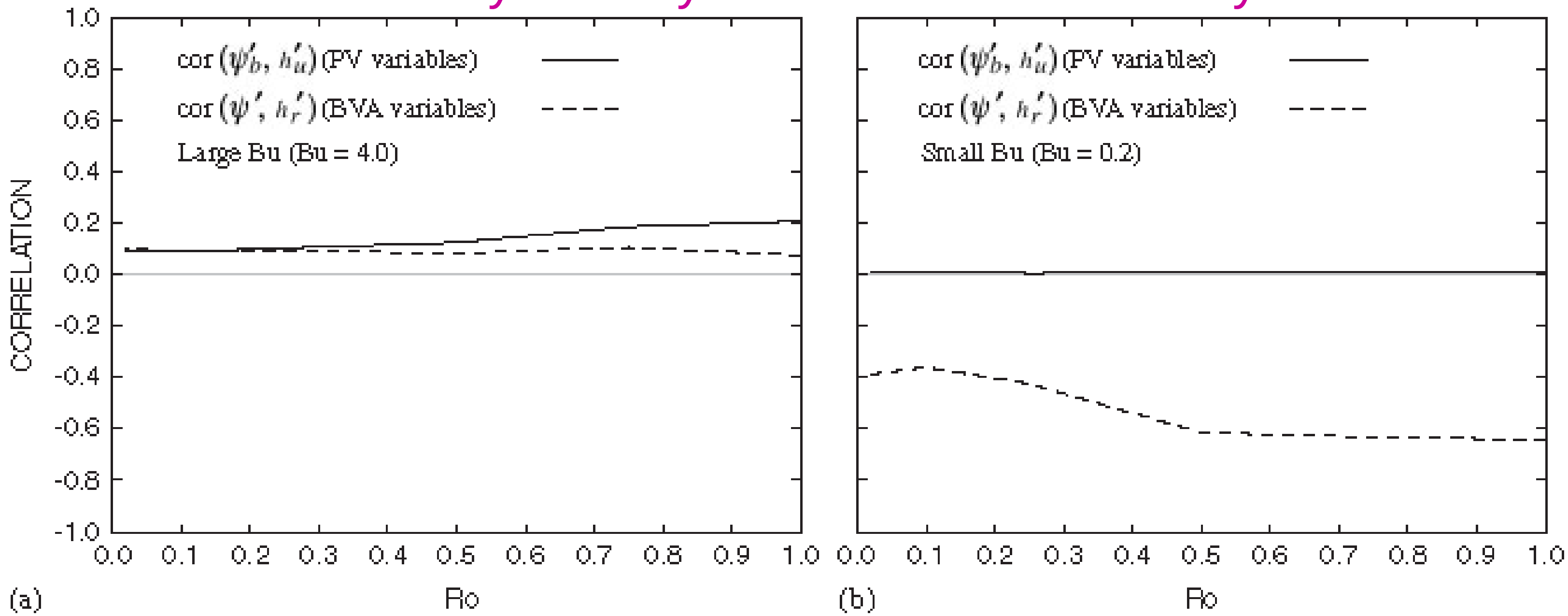
Bannister et al. (2008), Bannister and Cullen (2007)…

1. Balanced state preserves streamfunction. $\delta\Psi^b=\delta\Psi$
   - $L<<L_R$ (tropics, small hor scales, large vert scales)
   - ECMWF, NCEP, Met Office, Meteo-France, CMC, HIRLAM…
2. Balanced state preserves mass field.
   - $L>> L_R$ (large hor scales, small vert scales)
3. Balanced state preserves potential vorticity.
   - Accommodates different dynamical regimes
   - Allows for unbalanced vorticity as in Normal mode approach
   - Control variables only weakly correlated
   - Requires solving two elliptical eq simultaneously for $\delta\Psi^b$,δP^b
   - May be practical but more work is needed (Cullen 2002)

PV based control variables (Ψ^b, χ, h^u)

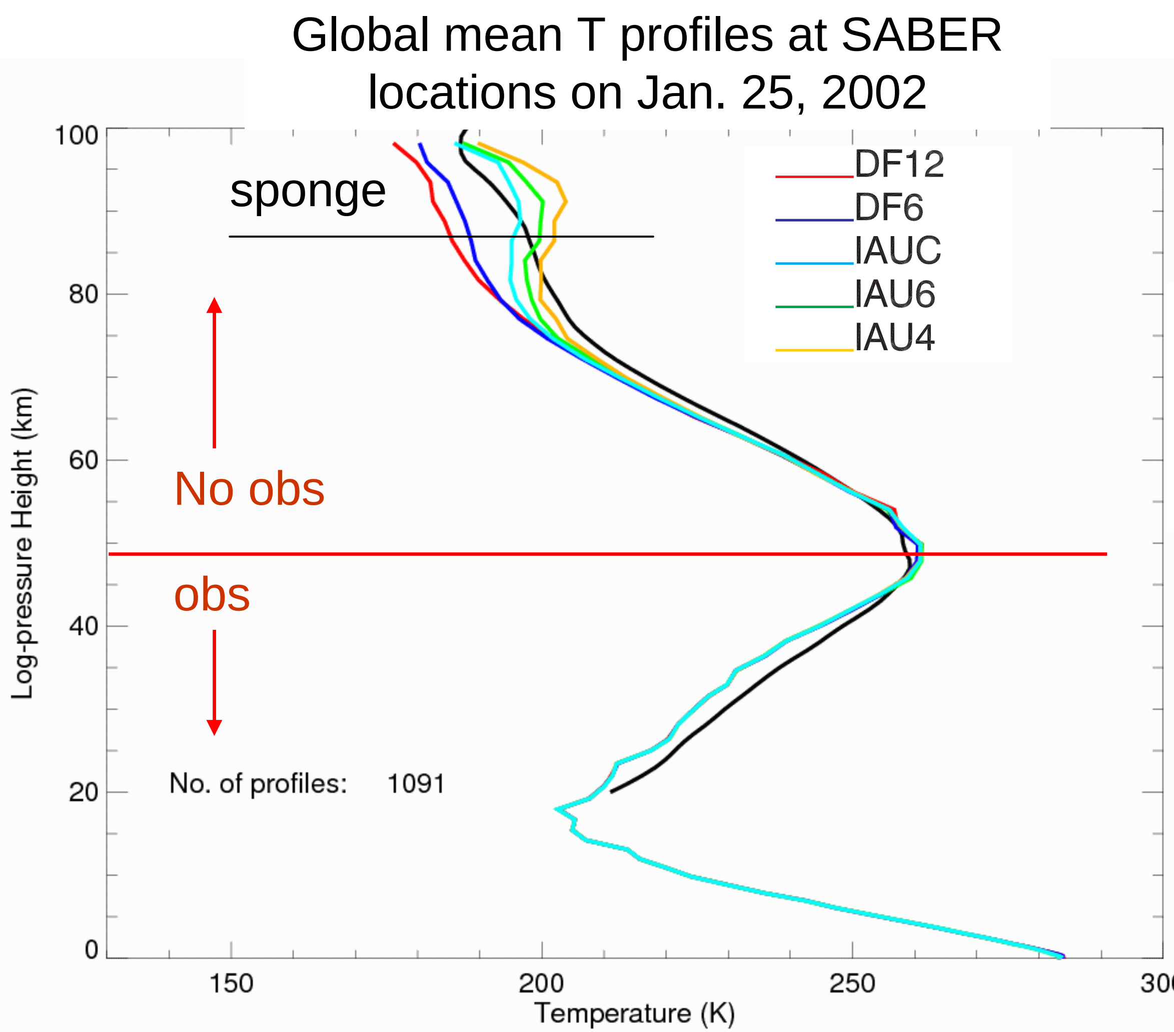
Correlation between balanced wind and unbalanced height errors for 1D shallow water equations

PV dominated by vorticity | PV dominated by mass



Bannister et al. (2008)

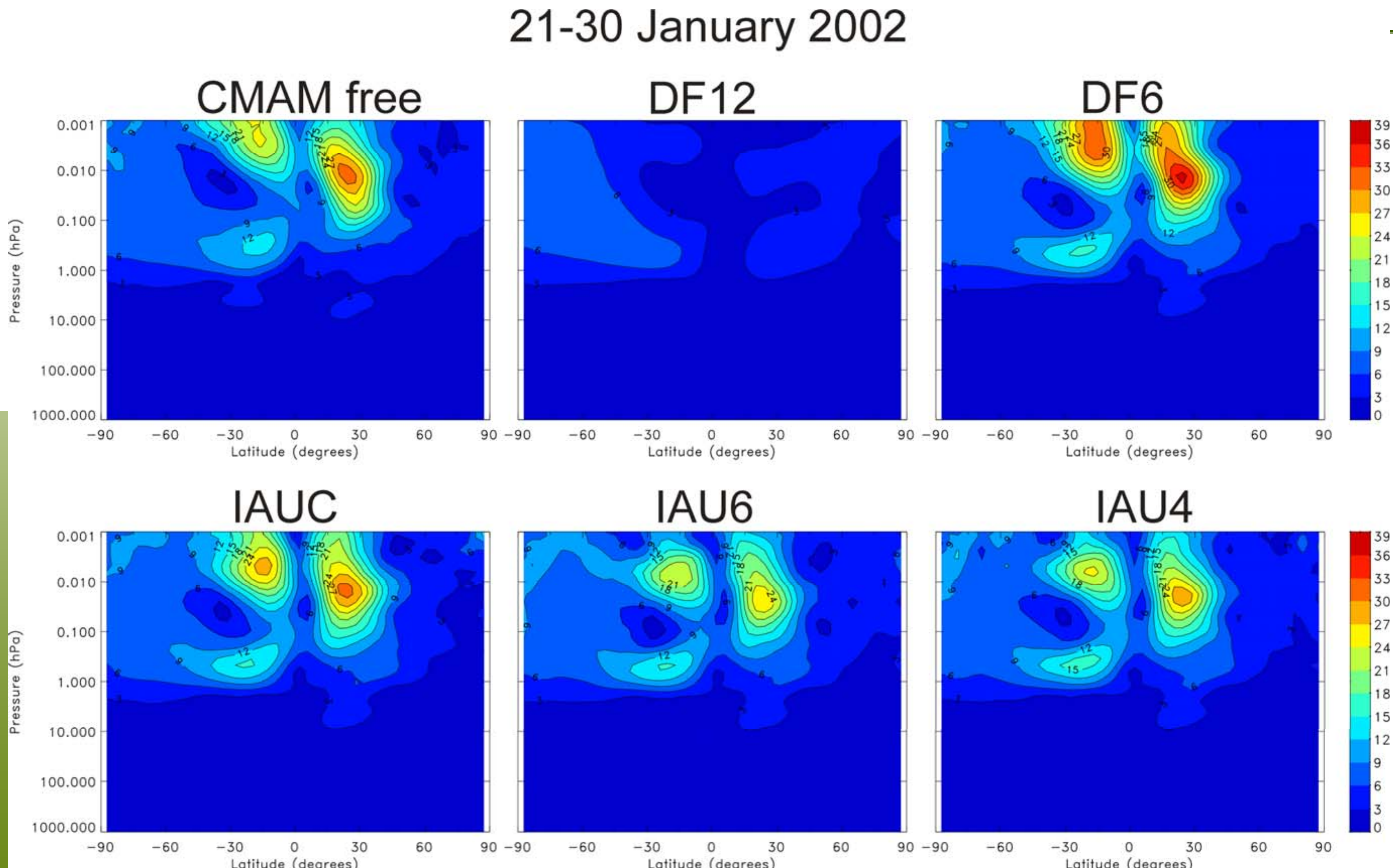
Insufficient filtering of spurious waves can lead to global temperature bias



More waves→ more damping→ more heating

Sankey et al. (2007)

Filtering scheme can enhance or wipe out the diurnal tide

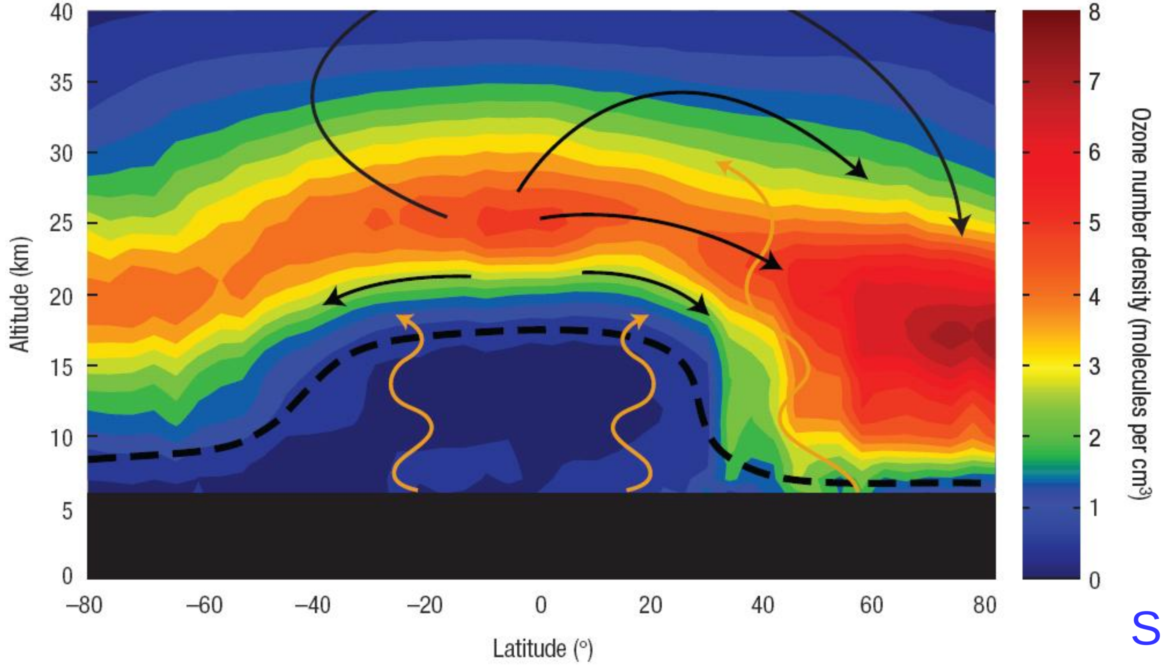


Sankey et al. (2007)

Implications for tracer transport

- Assimilated winds are often used to drive chemistry-transport models
- If the transport is well represented, then modeled species can be compared with observations to assess photochemical processes
- "…current DAS products will not give realistic trace gas distributions for long integrations" – Schoeberl et al. (2003)
- Vertical motion is noisy, horizontal motion is noisy in tropics. Leads to too rapid tracer transport (Weaver et al. 1993, Douglas et al. 2003, Schoeberl et al. 2003, …)

Stratospheric Age of air



Ozone from OSIRIS for March 2004

Shaw and Shepherd (2008)

- Measurements:
  - Use long-lived tracers with linear trends e.g. SF_6 or annual mean CO_2.

Assimilated winds produce much younger ages than GCM winds when used to drive CTMs

Note the weak latitudinal gradients

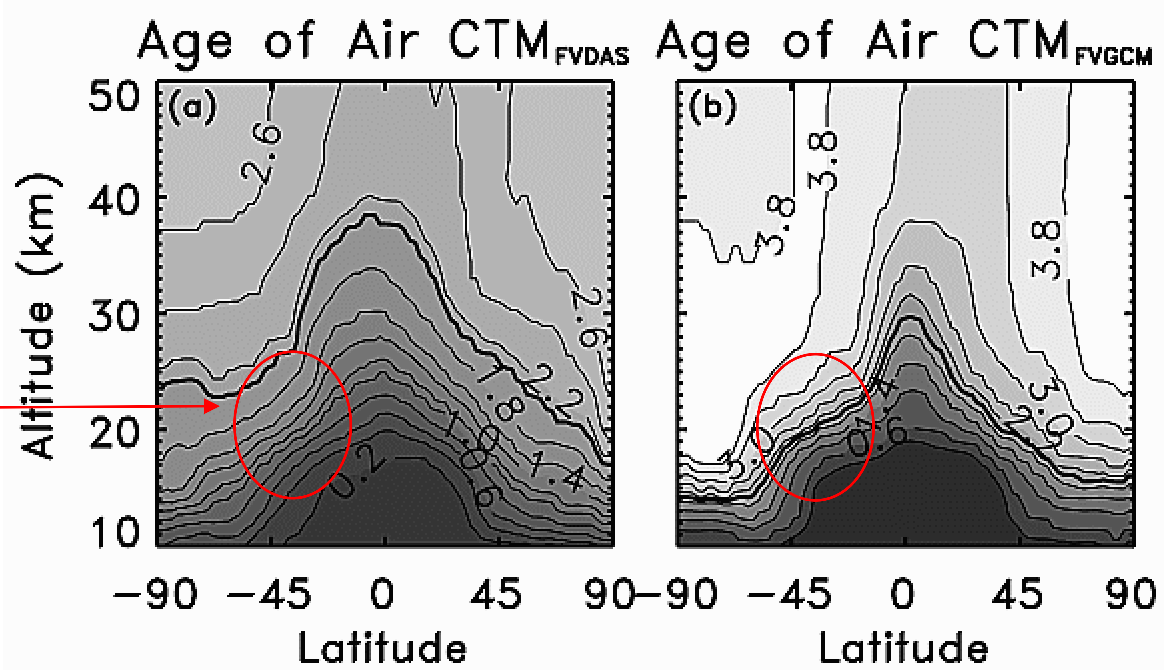


Figure 6. (a) Age of air (years) calculated from an SF-6 simulation using CTM_{FVDAS}. The age calculation converges after 5 years integration. (b) Same as Figure 6a but using CTM_{FVGCM}. The age calculation converges after 9 years integration. The contour interval is 0.2 years; the 2-year contour is bold for both panels.

Douglass et al. (2003)

The End

NNMI and balance constraints

Leith (1980) f-plane, Boussinesq (small vert scales)

	Order	0	1	2
Physical space	**Mass-wind**	Geostrophy $\nabla^2\Phi = f\nabla^2\Psi$	Nonlinear balance $\nabla^2\Phi = f\nabla^2\Psi + 2(\Psi_{xx}\Psi_{yy} - (\Psi_{xy})^2)$	
Physical space	**Divergence or vert velocity**	Non Divergence	QG omega eq.	
Normal mode approach	**Gravity modes**	$G(t=0)=0$	$\frac{dG}{dt}(t=0)=0$	$\frac{d^2G}{dt^2}(t=0)=0$
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Physical space

Normal mode approach

NNMI and balance constraints

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